

OMAJUHO JEMINE SPENCER
15/ENG06/054
MECHANICAL ENGINEERING
ENG 281: ASSIGNMENTS

i) $\frac{dy}{dt} + 3y = e^{-2t}$

$$y(t) + 3y(t) = e^{-2t}$$

$$L\{y'(t)\} = sY(s) - y(0)$$

$$L\{y(t)\} = Y(s)$$

$$L^{-1}\{e^{-2t}\} = \frac{1}{s+2}$$

$$sY(s) \cdot Y(s) + 3Y(s) = \frac{1}{s+2}$$

$$(s-3)Y(s) - 2 = \frac{1}{s+2}$$

$$(s-3)Y(s) = \frac{1}{s+2} + 2 = \frac{1+2s+4}{s+2} = \frac{2s+5}{s+2}$$

$$Y(s) = \frac{2s+5}{(s+2)(s-3)} = \frac{A}{s+2} + \frac{B}{s-3}$$

$$2(-2) + 5 = A(-2-3) \Rightarrow 7A = \frac{1}{5}$$

$$2(3) + 5 = B(3+2) \Rightarrow 7B = \frac{11}{5}$$

$$Y(s) = \frac{-1}{5(s+2)} + \frac{11}{5(s-3)} = \frac{-1}{5}e^{-2t} + \frac{11}{5}e^{3t}$$

ii) $3\frac{dy}{dt} - 6y = \sin 2t = 3y'(t) - 6y(t)$

$$L\{y'(t)\} = sY(s) - y(0)$$

$$L\{y(t)\} = Y(s)$$

$$L\{\sin 2t\} = \frac{2}{s^2+2^2} = \frac{2}{s^2+4}$$

$$3s + Y(s) - 3 + 6Y(s) - 6Y(s) = \frac{2}{s^2+4}$$

$$Y(s) [3s - 6] = \frac{2}{s^2+4} + 3 = \frac{2 + 3s^2 + 12}{s^2+4} = \frac{3s^2 + 14}{s^2+4}$$

$$Y(s) = \frac{3s^2 + 14}{(s-2)(s^2+4)} = \frac{A}{s-2} + \frac{B}{s^2+4}$$

$$3s^2 + 14 = A(s^2+4) + B(s-2)$$

$$4A - 6B = 14$$

$$B = -\frac{1}{3}$$

$$Y(s) = \frac{3}{3(s-2)} - \frac{1}{3(s^2+4)}$$

$$= e^{2t} - \frac{1}{6} \sin 2t$$

iii) $\frac{dy}{dt} - 4y = 8$

$$Y'(t) - 4y(t) = 8$$

$$Y'(t) - 4y(t) = 8$$

$$sY(s) - Y(s) - 4y(s) = \frac{8}{s}$$

$$(s-4)Y(s) = \frac{8}{s+2} = \frac{8}{s+2}$$

$$Y(s) = \frac{8}{s(s-4)}$$

$$\frac{2s+8}{s(s-4)} = \frac{A}{s} + \frac{B}{s-4}$$

$$2s+8 = A(s-4) + B(s)$$

$$2(4)+8 = A(4-4) + B(4) \Rightarrow 7B = 4$$

$$2(-4)+8 = A(-4-4) + B(-4) = -7A = 4 \Rightarrow A = -\frac{4}{7}$$

$$L^{-1}\left[-\frac{4}{7s} + \frac{4}{7(s-4)}\right] = -\frac{4}{7}e^{0t} + \frac{4}{7}e^{4t}$$

iv) $\frac{d^2y}{dt^2} - 2\frac{dy}{dt} + 5y = e^{2t}$

$$y''(t) - 2y'(t) + 5y(t) = e^{2t}$$

$$L\{y''(t)\} = s^2Y(s) - sy(0) - y'(0)$$

$$L\{y'(t)\} = sY(s) - y(0)$$

$$L\{e^{2t}\} = \frac{1}{s-2}$$

$$s^2Y(s) - sy(0) - y'(0) - 2[sY(s) - y(0)] + 5Y(s) = \frac{1}{s-2}$$

$$[s^2 - 2s + 5]Y(s) + (-2s + 2)Y(s) = \frac{1}{s-2}$$

$$Y(s) = \frac{1}{(s-2)(s^2-2s+5)}$$

$$= \frac{1}{s-2} + \frac{2s-7}{s^2-2s+5}$$

$$Y(s) = \frac{1}{s-2} + \frac{2s-7}{(s-2)(s^2-2s+5)}$$

$$2s^2 - 7s + 7 = \frac{A}{s-2} + \frac{B}{s^2-2s+5}$$

$$2s^2 - 7s + 7 = A(s^2 - 2s + 5) + B(s-2)$$

$$A = 2$$

$$B = -\frac{3}{2}$$

$$f(s) = \frac{2}{s-2} - \frac{3}{s-2+s}$$

$$= 2e^{2t} - \frac{1}{3} + \ln 2t$$

$$v) \frac{d^2 y}{dt^2} - 6 \frac{dy}{dt} + 8y = e^{3t}$$

$$s^2 Y(s) - 6sY(s) - 4Y(s) - 6sY(s) + 6Y(s) + 8Y(s) = \frac{1}{s-3}$$

$$[s^2 - 6s + 8] Y(s) = \frac{1}{s-3} + 2 = \frac{1+2s-6}{s-3}$$

$$= \frac{2s-5}{s-3}$$

$$Y(s) = \frac{2s-5}{(s-3)(s-2)(s-4)} = \frac{A}{s-3} + \frac{B}{s-2} + \frac{C}{s-4}$$

$$2s-5 = A[(s-2)(s-4)] + B[(s-3)(s-4)] + C[(s-3)(s-2)]$$

$$2s-5 = A[s^2 - 6s + 8] + B[s^2 - 7s + 2] + C[s^2 - 5s + 6]$$

$$2(3) - 5 = A(3-2)(3-4) = 7A = 4$$

$$2(4) - 5 = C(4-3)(4-2) = 7C = 3 \Rightarrow C = \frac{3}{7}$$

$$-6A - 7B + 5C = 7$$

$$-6C - 7B - 5(\frac{3}{7}) = 2$$

$$-7B = 2 + 15/2 - 6 = \frac{4+15-12}{2} = \frac{7}{2} \Rightarrow B = -\frac{1}{2}$$

$$L^{-1} \left[\frac{1}{s-3} - \frac{1}{2(s-2)} + \frac{3}{2(s-4)} \right]$$

$$= e^{3t} - \frac{1}{2} e^{2t} + \frac{3}{2} e^{4t}$$